Surface charge density of the insulated sphere :-

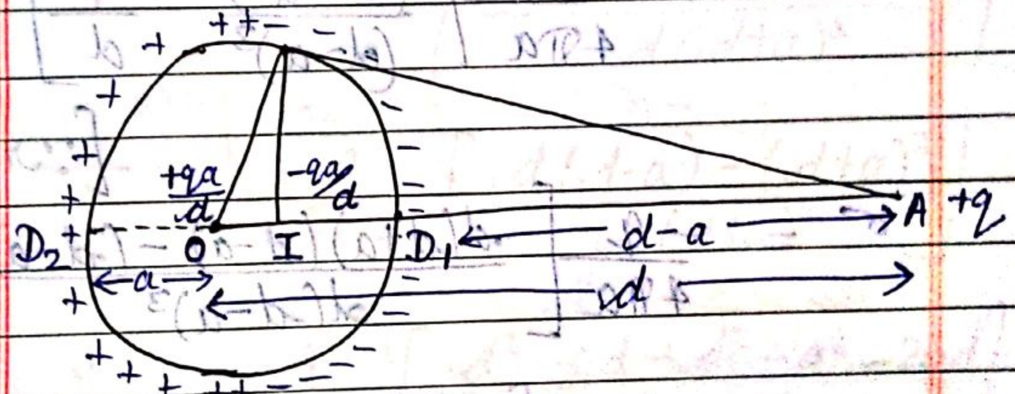
From the definition of surface charge density, the charge ~~per~~ per unit area is known as its surface charge density.

Now, Surface charge density = $\frac{\text{charge}}{\text{Area}}$

Now, the surface charge density due to the image charge $(+q\,a/d)$ at the centre of sphere) can be expressed as

$$\sigma_1 = \frac{qa}{d\,4\pi a^2} = \frac{q}{4\pi a d} \quad (4)$$

Now, the surface charge density due to the point charge $+q$ and image charge $-qa/d$ at external point can be expressed as :-



$$\sigma_2 = -\frac{(d^2-a^2)q}{4\pi a r^3} \quad (5)$$

Now, the total surface charge density of insulated sphere can be expressed as

$$\sigma = \sigma_1 + \sigma_2$$

$$\sigma = \frac{q}{4\pi a}\left[\frac{1}{d} - \frac{(d^2-a^2)}{r^3}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{(d^2-a^2)}{r_1^3} - \frac{1}{d}\right] \quad \text{---(6)}$$

Now, if the point charge is connected with the first boundary of the sphere i.e; D_1 then total conductivity can be expressed as

$$\sigma = \frac{-q}{4\pi a}\left[\frac{(d^2-a^2)}{(d-a)^3} - \frac{1}{d}\right]$$

$[\because r = d-a]$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{d(d+a)(d-a) - (d-a)^3}{d(d-a)^3}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{d(d+a) - (d-a)^2}{d(d-a)^2}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{d^2+da-d^2-a^2+2ad}{d(d-a)^2}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{3ad-a^2}{d(d-a)^2}\right]$$

$$\sigma = \frac{-q}{4\pi}\left[\frac{3d-a}{d(d-a)^2}\right] \quad \text{---(7)}$$

$$\boxed{\sigma = -ve} \quad \text{—— (7)}$$

If the pt. charge is connected with second boundary of sphere i.e., D_2 then conductivity can be expressed as:

$$\sigma = \frac{-q}{4\pi a}\left[\frac{(d^2-a^2)}{(d+a)^3} - \frac{1}{d}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{d(d^2-a^2) - (d+a)^3}{d(d+a)^3}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{d(d-a) - (d+a)^2}{d(d+a)^2}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{d^2 - ad - d^2 - a^2 - 2ad}{d(d+a)^2}\right]$$

$$\sigma = \frac{-q}{4\pi a}\left[\frac{-3ad - a^2}{d(d+a)^2}\right]$$

$$\sigma = \frac{q}{4\pi a}\left[\frac{3ad + a^2}{d(d+a)^2}\right] = \frac{q}{4\pi}\left[\frac{3d+a}{d(d+a)^2}\right]$$

$$\boxed{\sigma = +ve} \quad \text{—— (8)}$$

From eq^n (7) and (8), it is clear

that the surface charge density at I boundary remains -ve while at II boundary the surface charge density remains +ve.

Hence, the total density of the insulated sphere remains zero. Then from eq^n ⑥, we get

$$0 = \frac{-q}{4\pi a}\left[\frac{d^2-a^2}{r^3} - \frac{1}{d}\right]$$

$$\frac{d^2-a^2}{r^3} = \frac{1}{d} \Rightarrow \boxed{d(d^2-a^2) = r^3}$$

This is the required eq^n for insulated sphere & also known as line of <u>nil electrification</u>.

ii) <u>Force</u> of <u>charges</u> <u>in the</u> <u>insulated sphere</u> :-

From the def^n of coulomb's force we know that

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$